EXERCISE-1

SECTION (A) : EQUATION OF TRAVELLING WAVE (INCLUDING SINE WAVE)

A 1. The wave function for a traveling wave on a taut string is (in SI units)

$s(x, t) = (0.350\text{ m}) \sin (10\pi t - 3\pi x + \pi/4)$

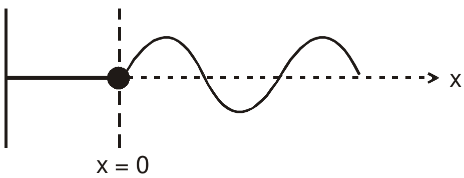
(a) What are the speed and direction of travel of the wave ?

(b) What is the vertical displacement of the string at t = 0, x = 0.100 m ?

(c) What are wavelength and frequency of the wave ?

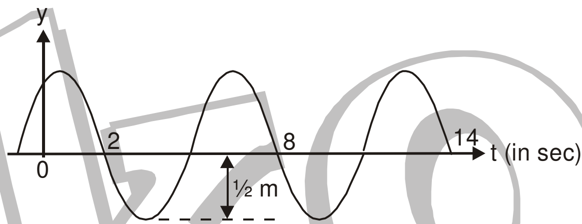
(d) What is the maximum magnitude of the transverse speed of the string?

A 2. The string shown in figure is driven at a frequency of 5.00 Hz. The amplitude of the motion is 12.0 cm, and the wave speed is 20.0 m/s. Furthermore, the wave is such that y = 0 at x = 0 and t = 0. Determine (a) the angular frequency and (b) wave number for this wave. (c) Write an expression for the wave function. Calculate (d) the maximum transverse speed and (e) the maximum transverse acceleration of a point on the string.

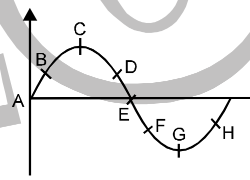


A 3. (a) Write the expression for y as a function of x and t for a sinusoidal wave traveling along a rope in the negative x direction with the following characteristics : A = 8.00 cm, λ = 80.0 cm, f = 3.00 Hz, and y(0, t) = 0 at t = 0. (b) Write the expression for y as a function of x and t for the wave in part (a) assuming that y(x, 0) = 0 at the point x = 10.0 cm.

A 4. The sketch in the figure shows displacement time curve of a sinusoidal wave at x = 8 m. Taking velocity of wave v = 6m/s along postive x-axis, write the equation of the wave.



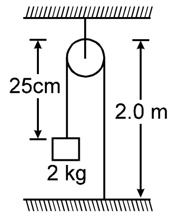
A 5. A transverse wave is travelling along a string from left to right. The fig. represents the shape of the string (snap-shot) at a given instant. At this instant (a) which points have an upward velocity (b) which points will have downward velocity (c) which points have zero velocity (d) which points have maximum magnitude of velocity.



SECTION (B) : SPEED OF A WAVE ON A STRING

B 1. A piano string having a mass per unit length equal to 5.00×10^{-3} kg/m is under a tension of 1350 N. Find the speed with which a wave travels on this string.

B 2. In the arrangement shown in figure, the string has mass of 4.5 g. How much time will it take for a transverse disturbance produced at the floor to reach the pulley ? Take g = 10 m/s^2.



SECTION (C) : POWER TRANSMITTED ALONG THE STRING

C 1. A 6.00 m segment of a long string has a mass of 180 g. A high-speed photograph shows that the segment contains four complete cycles of a wave. The string is vibrating sinusoidally with a frequency of 50.0 Hz and a peak-to-valley displacement of 15.0 cm. (The "peak-to-valley" displacement is the vertical distance from the farthest positive displacement to the farthest negative displacement.) (a) Write the function that describes this wave traveling in the positive x direction. (b) Determine the power being supplied to the string.

C 2. A 200 Hz wave with amplitude 1 mm travels on a long string of linear mass density 6 g/m kept under a tension of 60 N. (a) Find the average power transmitted across a given point on the string. (b) Find the total energy associated with the wave in a 2.0 m long portion of the string.

SECTION (D) : INTERFERENCE, REFLECTION, TRANSMISSION

D 1. A series of pulses, each of amplitude 0.150 m, are send down a string that is attached to a post at one end. The pulses are reflected at the post and travel back along the string without loss of amplitude. When two waves are present on the same string. The net displacement of a give point is the sum of the displacements of the individual waves at the point. What is the net displacement at point on the string where two pulses are crossing, (a) if the string is rigidly attached to the post ? (b) If the end at which reflection occurs is free to slide up and down?

D 2. Two identical traveling waves, moving in the same direction are out of phase by $\pi/2$ rad. What is the amplitude of the resultant wave in terms of the common amplitude y_m of the two combining waves?

D 3. Two waves are described by

$y_1 = 0.30 \sin [\pi(5x - 200)t]$

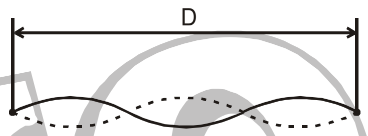
and $y_2 = 0.30 \sin [\pi(5x - 200t) + \pi/3]$

where y_1, y_2 and x are in meters and t is in seconds. When these two waves are combined, a traveling wave is produced. What are the (a) amplitude, (b) wave speed, and (c) wave length of that traveling wave ?

SECTION (E) : STANDING WAVES AND RESONANCE

E 1. What are (a) the lowest frequency, (b) the second lowest frequency, and (c) the third lowest frequency for standing waves on a wire that is 10.0 m long has a mass of 100 g. and is stretched under a tension of 250 N?

E 2. A nylon guitar string has a linear density of 7.20 g/m and is under a tension of 150 N. The fixed supports are distance D = 90.0 cm apart. The string is oscillating in the standing wave pattern shown in figure. Calculate the (a) speed. (b) wavelength, and (c) frequency of the traveling waves whose superposition gives this standing wave.



E 3. A string that is stretched between fixed supports separated by 75.0 cm has resonant frequencies of 420 and 315 Hz with no intermediate resonant frequencies. What are (a) the lowest resonant frequencies and (b) the wave speed?

E 4. A string oscillates according to the equation

$$y' = (0.50 \text{ cm}) \sin \left[\left(\frac{\pi}{3}\text{cm}^{-1}\right)x\right] \cos [(40\,\pi \text{ s}^{-1})t].$$

What are the (a) amplitude and (b) speed of the two waves (identical except for direction of travel) whose superposition gives this oscillation? (c) what is the distance between nodes? (d) What is the transverse speed of a particle of the string at the position x = 1.5 cm when $t = \frac{9}{8}$ s ?

E 5. In an experiment of standing waves, a string 90 cm long is attached to the prong of an electrically driven tuning fork that oscillates perpendicular to the length of the string at a frequency of 60 Hz. The mass of the string is 0.044 kg. What tension must the string be under (weights are attached to the other end) if it is to oscillate in four loops?

E 6. A string vibrates in 4 loops with a frequency of 400 Hz .
(a) What is its fundamental frequency ?
(b) What frequency will cause it to vibrate into 7 loops .

E 7. The vibration of a string of length 60 cm is represented by the equation,
$y = 3 \cos (\pi x/20) \cos (72\pi t)$ where x & y are in cm and t in sec.
(i) Write down the component waves whose superposition gives the above wave.
(ii) Where are the nodes and antinodes located along the string.
(iii) What is the velocity of the particle of the string at the position x = 5 cm & t = 0.25 sec.

E 8. A string fixed at both ends is vibrating in the lowest mode of vibration for which a point at quarter of its length from one end is a point of maximum displacement. The frequency of vibration in this mode is 100 Hz . What will be the frequency emitted when it vibrates in the next mode such that this point is again a point of maximum displacement.

EXERCISE-2

SECTION (A) : EQUATION OF TRAVELLING WAVE (INCLUDING SINE WAVE)

A 1*. A wave equation which gives the displacement along the Y direction is given by

$$y = 10^{-4} \sin (60t + 2x)$$

where x and y are in metres and t is time in seconds. This represents a wave

(A) travelling with a velocity of 30 m/s in the negative x direction
(B) of wavelength π metre
(C) of frequency $30/\pi$ hertz
(D) of amplitude 10^{-4} metre travelling along the negative x direction.

A 2. A transverse wave is described by the equation $Y = Y_0 \sin 2\pi (ft - x/\lambda)$. The maximum particle velocity is equal to four times the wave velocity if

(A) $\lambda = \pi Y_0/4$ (B) $\lambda = \pi Y_0/2$ (C) $\lambda = \pi Y_0$ (D) $\lambda = 2\pi Y_0$

A 3*. The displacement of particles in a string stretched in x–direction is represented by y . Among the following expressions for y , those describing wave motion are :

(A) $\cos (kx) \sin(\omega t)$ (B) $k^2x^2 - \omega^2t^2$ (C) $\cos^2 (kx + \omega t)$ (D) $\cos (k^2x^2 - \omega^2t^2)$

A 4. A transverse wave of amplitude 0.50m, wavelength 1m and frequency 2 hertz is propagating in a string in the negative x-direction. The expression from of the wave is

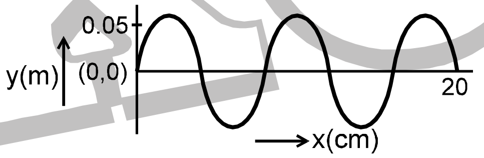
(A) $y(x, t) = 0.5 \sin (2\pi x - 4\pi t)$ (B) $y(x, t) = 0.5 \cos (2\pi x + 4\pi t)$
(C) $y(x, t) = 0.5 \sin (\pi x - 2\pi t)$ (D) $y(x, t) = 0.5 \cos (2\pi x - 2\pi t)$

A 5. A travelling wave on a string is given by $y = A \sin [\alpha x + \beta t + \frac{\pi}{6}]$. The displacement and velocity of oscillation of a point $\alpha = 0.56$ /cm, $\beta = 12$/sec, A = 7.5 cm, x = 1 cm and t = 1s is

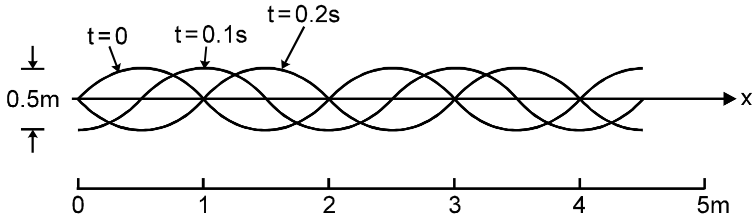
(A) 4.6 cm, 46.5 cm s^{-1} (B) 3.75 cm, 77.94 cm s^{-1}
(C) 1.76 cm, 7.5 cm s^{-1} (D) 7.5 cm, 75 cm s^{-1}

A 6. For the wave shown in figure, the equation for the wave, travelling along +x axis with velocity 350 ms^{-1} when its position at t = 0 is as shown

(A) 0.05 sin (78.5x – 27500 t)
(B) 0.05 sin (75.8x – 27000 t)
(C) 1 sin (78.5x – 27500 t)
(D) 0.05 sin (57.8x + 25700 t)



A 7. Three consecutive flash photographs of a travelling wave on a string are reproduced in the figure here. The following observations are made. Mark the one which is correct.
(Mass per unit length of the string = 3 g/cm.)



(A) displacement amplitude of the wave is 0.25 m, wavelength is 1 m, wave speed is 2.5 m/s and the frequency of the driving force is 0.2/s.
(B) displacement amplitude of the wave is 2.0 m, wavelength is 2 m, wave speed is 0.4 m/s and the frequency of the driving force is 0.7/s.
(C) displacement amplitude of the wave is 0.25 m, wavelength is 2 m, wave speed is 5 m/s and the frequency of the driving force is 2.5 /s.
(D) displacement amplitude of the wave is 0.5 m, wavelength is 2 m, wave speed is 2.5 m/s and the frequency of the driving force is 0.2/s.

A 8. The amplitude of a wave disturbance propagating in the positive x-direction is given by $y = 1/(1 + x^2)$ at time t = 0 and $y = 1/[1 + (x - 1)^2]$ at t = 2 seconds where x and y are in metres. The shape of the wave disturbance does not change during the propagation. The velocity of the wave is

(A) 2.5 m/s (B) 0.25 m/s (C) 0.5 m/s (D) 5 m/s

A 9. Two stretched wires A and B of the same lengths vibrate independently. If the radius, density and tension of wire A are respectively twice those of wire B, then the frequency of vibration of A relative to that of B is
(A) 1 : 1 (B) 1 : 2 (C) 1 : 4 (D) 1 : 8

SECTION (B) : POWER TRANSMITTED ALONG THE STRING

B 1. For a wave displacement amplitude is 10^{-8} m, density of air 1.3 kg m^{-3}, velocity in air 340 ms^{-1} and frequency is 2000 Hz. The intensity of wave is
(A) 5.3×10^{-4} Wm^{-2} (B) 5.3×10^{-6} Wm^{-2} (C) 3.5×10^{-8} Wm^{-2} (D) 3.5×10^{-6} Wm^{-2}

B 2. A sinusoidal wave with amplitude y_m is travelling with speed V on a string with linear density ρ. The angular frequency of the wave is ω. The following conclusions are drawn. Mark the one which is correct.
(A) doubling the frequency doubles the rate at which energy is carried along the string
(B) if the amplitude were doubled, the rate at which energy is carried would be halved
(C) if the amplitude were doubled, the rate at which energy is carried would be halved
(D) the rate at which energy is carried is directly proportional to the velocity of the wave.

B 3. Sinusoidal waves 5.00 cm in amplitude are to be transmitted along a string having a linear mass density equal to 4.00×10^{-2} kg/m. If the source can deliver a maximum power of 90 W and the string is under a tension of 100 N, then the highest frequency at which the source can operate is (take $\pi^2 = 10$):
(A) 45.3 Hz (B) 50 Hz (C) 30 Hz (D) 62.3 Hz

B 4. A wave moving with constant speed on a uniform string passes the point x = 0 with amplitude A_0, angular frequency ω_0 and average rate of energy transfer P_0. As the wave travels down the string it gradually loses energy and at the point x = ℓ, the average rate of energy transfer becomes $\frac{P_0}{2}$. At the point x = ℓ, angular frequency and amplitude are respectively :
(A) ω_0 and $A_0/\sqrt{2}$ (B) $\omega_0/\sqrt{2}$ and A_0 (C) less than ω_0 and A_0 (D) $\omega_0/\sqrt{2}$ and $A_0/\sqrt{2}$

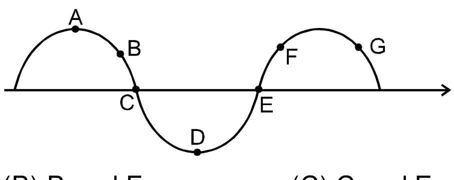
SECTION (C) : INTERFERENCE, REFLECTION, TRANSMISSION

C 1. When two waves of the same amplitude and frequency but having a phase difference of ϕ, travelling with the same speed in the same direction (positive x), interfere, then
(A) their resultant amplitude will be twice that of a single wave but the frequency will be same
(B) their resultant amplitude and frequency will both be twice that of a single wave
(C) their resultant amplitude will depend on the phase angle while the frequency will be the same
(D) the frequency and amplitude of the resultant wave will depend upon the phase angle.

C 2. The rate of transfer of energy in a wave depends
(A) directly on the square of the wave amplitude and square of the wave frequency
(B) directly on the square of the wave amplitude and square root of the wave frequency
(C) directly on the wave frequency and square of the wave amplitude
(D) directly on the wave amplitude and square of the wave frequency

C 3. The effects are produced at a given point in space by two waves described by the equations, $y_1 = y_m \sin \omega t$ and $y_2 = y_m \sin (\omega t + \phi)$ where y_m is the same for both the waves and ϕ is a phase angle. Tick the incorrect statement among the following.
(A) the maximum intensity that can be achieved at a point is twice the intensity of either wave and occurs if $\phi = 0$
(B) the maximum intensity that can be achieved at a point is four times the intensity of either wave and occurs if $\phi = 0$
(C) the maximum amplitude that can be achieved at the point its twice the amplitude of either wave and occurs at $\phi = 0$
(D) When the intensity is zero, the net amplitude is zero, and at this point $\phi = \pi$.

C 4. The following figure depicts a wave travelling in a medium. Which pair of particles are in phase.



(A) A and D (B) B and F (C) C and E (D) B and G

SECTION (D) : STANDING WAVES AND RESONANCE

D 1. A wave represented by the equation $y = a\cos(kx - \omega t)$ is superposed with another wave to form a stationary wave such that the point $x = 0$ is a node. The equation for other wave is :
(A) $a\sin(kx + \omega t)$ (B) $-a\cos(kx + \omega t)$ (C) $-a\cos(kx - \omega t)$ (D) $-a\sin(kx - \omega t)$

D 2. In a stationary wave, the distance between a consecutive node and antinode is -
(A) 2λ (B) $\frac{\lambda}{4}$ (C) λ (D) $\frac{\lambda}{2}$

D 3. A stretched sonometer wire resonates at a frequency of 350 Hz and at the next higher frequency of 420 Hz. The fundamental frequency of this wire is
(A) 350 Hz (B) 5 Hz (C) 70 Hz (D) 170 Hz

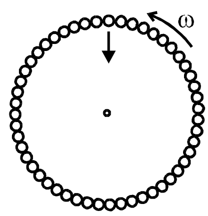
D 4. On a stretched string the waves of the form, $y_1 = A\sin(\omega t - kx)$ and $y_2 = -A\sin(\omega t + kx)$ are superimposed. The following conclusions are drawn about the resultant waveform. Mark the one which is incorrect.
(A) the shape of the string at each point is a sine curve whose amplitude varies with time
(B) the appearance is not that of a travelling wave shape but of a sinusoidal displacement in one position which grows larger and smaller with time
(C) each point in the string still undergoes simple harmonic motion but instead of the progressively increasing phase difference between motions of adjacent points, all points move in phase or 180° out of phase
(D) in the resultant wave each particle of the string vibrates with the same amplitude.

EXERCISE-3

1. One end of two wires of the same metal and of same length (with radius, r and 2r) are joined together. The wire is used as sonometer wire and the junction is placed in between two bridges. The tension T is applied to the wire. If at a junction a node is formed then the ratio of number of loops formed in the wires will be:
(A) 1 : 2 (B) 2 : 3 (C) 3 : 4 (D) 4 : 5

2*. The particle displacement in a wave is given by
$y = 0.2 \times 10^{-5}\cos(500\,t - 0.025\,x)$
where the distances are measured in meters and time in seconds. Now
(A) wave velocity is 2 × 104 ms^{-1} (B) particle velocity is $2 \times 10^4\ ms^{-1}$
(C) initial phase difference is $\frac{\pi}{2}$ (D) wavelength of the wave is (80π) m

3. A circular loop of rope of length L rotates with uniform angular velocity ω about an axis through its centre on a horizontal smooth platform. Velocity of pulse produced due to slight radial displacement is given by

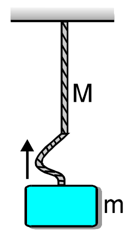


(A) ωL (B) $\frac{\omega L}{2\pi}$ (C) $\frac{\omega L}{\pi}$ (D) $\frac{\omega L}{4\pi^2}$

4. Two wires of the same material and radii r and 2r are welded together end to end. The combination is used as a sonometer wire and kept under tension T. The welded point is mid-way between the two bridges. When stationary waves are set up in the composite wire, the joint is a node. Then the ratio of the number of loops formed in the thinner to thicker wire is
(A) 2 : 3 (B) 1 : 2 (C) 2 : 1 (D) 5 : 4

5. Three waves of equal frequency having amplitudes 10 μm, 4 μm and 7 μm arrive at a given point with a successive phase difference of $\pi/2$. The amplitude of the resulting wave is μm in given by
(A) 7 (B) 6 (C) 5 (D) 4

6. A uniform rope of length ℓ and mass M hangs vertically from a rigid support. A block of mass m is attached to the free end of the rope. A transverse pulse of wavelength λ is produced at the lower end of the rope. The wavelength of the pulse, when it reaches the top of the rope, is



(A) $\lambda\sqrt{\frac{M-m}{m}}$ (B) $\lambda\frac{M+m}{m}$ (C) $\lambda\sqrt{\frac{m}{M+m}}$ (D) $\lambda\sqrt{\frac{M+m}{m}}$

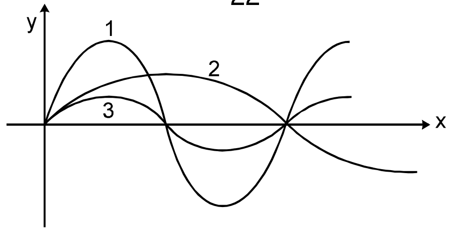
7. A steel wire of length 1 m and mass 0.1 kg and having a uniform cross-sectional area of 10^{-6} m² is

rigidly fixed at both ends. The temperature of the wire is lowered by 20°C. If the transverse waves are set up by plucking the string in the middle, the frequency of the fundamental note of vibration is ($Y_{steel} = 2 \times 10^{11}$ N/m², $\alpha_{steel} = 1.21 \times 10^{-5}$/°C)
(A) 44 Hz (B) 88 Hz (C) 22 Hz (D) 11 Hz

8. A stone is hung in air from a wire which is stretched over a sonometer. The bridges of the sonometer are 40 cm apart when the wire is in unison with a tuning fork of frequency 256. When the stone is completely immersed in water, the length between the bridges is 22 cm for re-establishing unison. The specific gravity of the material of the stone is:

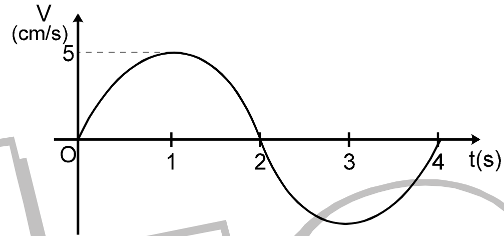
(A) $\frac{(40)^2}{(40)^2+(22)^2}$ (B) $\frac{(40)^2}{(40)^2-(22)^2}$ (C) $256\times\frac{22}{40}$ (D) $256\times\frac{40}{22}$

9. Graph shows three waves that are separately sent along a string that is stretched under a certain tension along x–axis. If ω_1,ω_2 and ω_3 are their angular frequencies respectively then



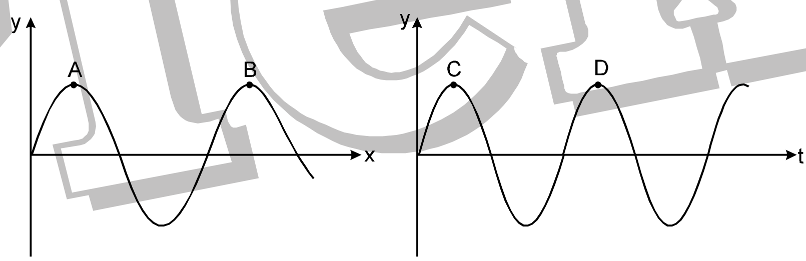
(A) $\omega_1 = \omega_3 > \omega_2$ (B) $\omega_1 > \omega_2 > \omega_3$
(C) $\omega_2 > \omega_1 = \omega_3$ (D) $\omega_1 = \omega_2 = \omega_3$

10. A certain transverse sinusoidal wave of wavelength 20 cm is moving in the positive x direction. The transverse velocity of the particle at x = 0 as a function of time is shown. The amplitude of the motion is :



(A) $\frac{5}{\pi}$ cm (B) $\frac{\pi}{2}$ cm
(C) $\frac{10}{\pi}$ cm (D) 2π cm

11. The same progressive wave is represented by two graphs I and II. Graph I shows how the displacement 'y' varies with the distance x along the wave at a given time. Graph II shows how y varies with time t at a given point on the wave. The ratio of measurements AB to CD, marked on the curves, represents :



(I) (II)

(A) wave number k (B) wave speed V.
(C) frequency ν. (D) angular frequency ω.

12. A transverse periodic wave on a string with a linear mass density of 0.200 kg/m is described by the following equation
$$y = 0.05 \sin(420t - 21.0\,x)$$
where x and y are in metres and t is in seconds.
The tension in the string is equal to :
(A) 32 N (B) 42 N (C) 66 N (D) 80 N

13. Equation of a standing wave is generally expressed as $y = 2A \sin\omega t \cos kx$. In the equation, quantity ω/k represents
(A) the transverse speed of the particles of the string.
(B) the speed of either of the component waves.
(C) the speed of the standing wave.
(D) a quantity that is independent of the properties of the string.

14. A 20 cm long rubber string obeys Hook's law. Initially when it is stretched to make its total length of

24 cm, the lowest frequency of resonance is υ_0. It is further stretched to make its total length of 26 cm. The lowest frequency of resonance will now be :
(A) the same as υ_0 (B) greater than υ_0 (C) lower than υ_0 (D) None of these

15. The wave–function for a certain standing wave on a string fixed at both ends is $y(x,t) = 0.5 \sin (0.025\pi x) \cos 500t$ where x and y are in centimeters and t is in seconds. The shortest possible length of the string is:
(A) 126 cm (B) 160 cm (C) 40 cm (D) 80 cm

16. A 75 cm string fixed at both ends produces resonant frequencies 384 Hz and 288 Hz without there being any other resonant frequency between these two. Wave speed for the string is :
(A) 144 m/s (B) 216 m/s (C) 108 m/s (D) 72 m/s

17. A string of length 'ℓ' is fixed at both ends. It is vibrating in its 3rd overtone with maximum amplitude 'a'. The amplitude at a distance $\frac{\ell}{3}$ from one end is :
(A) a (B) 0 (C) $\frac{\sqrt{3}a}{2}$ (D) $\frac{a}{2}$

EXERCISE-4

1. A transverse sinusoidal wave is generated at one end of a long, horizontal string by a bar that moves up and down through a distance of 1.00 cm . The motion is continuous and is repeated regularly 120 times per second. The string has linear density 90 gm/m and is kept under a tension of 900 N. Find :
(a) the maximum value of the transverse speed u
(b) the maximum value of the transverse component of the tension
(c) What is the transverse displacement y when this maximum value of the tension occurs ?
(d) What is the maximum power transferred along the string .
(e) What is the transverse displacement y when this maximum power transfer occur
(f) What is the minimum power transfer along the string
(g) What is the transverse displacement y when the minimum power transfer occurs
[Leave the answer in terms of π wherever it occurs]

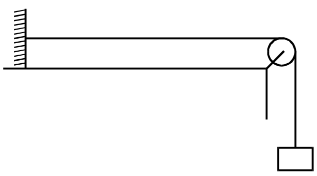
2. A standing wave is produced in a steel wire of mass 100 gm tied to two fixed supports. The length of the string is 2 m & strain in it is 0.4 %. The string vibrates in four loops. Assuming one end of the string to be at x = 0, all particles to be at rest at t = 0 and maximum amplitude to be 3 mm, find :
(a) Wavelength & frequency of the wave .
(b) Equation of the standing wave .
(c) Equation of the travelling waves whose superposition is the given standing wave. Also find the velocity of these travelling waves .
(d) Maximum kinetic energy of the wire . [$\pi^2 = 10$]
[Given : density of steel $=4 \times 10^3$ kg/m^3, young's modulus of steel $=1.6 \times 10^{11}$ N/m^2]

3. A string 120 cm in length and fixed at both ends sustains a standing wave, with the points of the string at which the displacement amplitude is equal to 3.5 mm being separated by 15.0 cm. Find the maximum displacement amplitude. To which overtone do these oscillations correspond ?

4. A steel wire of length $50\sqrt{3}$ cm is connected to an aluminium wire of length 60 cm and stretched between two fixed supports. The tension produced is 104 N, if the cross section area of each wire is 1mm^2. If a transverse wave is set up in the wire, find the lowest frequency for which standing waves with node at the joint are produced.
(density of aluminium = 2.6 gm/cm^3 and density of steel = 7.8 gm/cm^3)

5. Three resonant frequencies of a string are 90, 150 and 210 Hz. (a) Find the highest possible fundamental frequency of vibration of this string. (b) Which harmonics of the fundamental are the given frequencies ? (c) Which overtone are these frequencies. (d) If the length of the string is 80 cm, what would be the speed of a transverse wave on this string ?

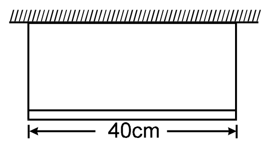
6. Figure shows a string stretched by a block going over a pulley. The string vibrates in is tength harmonic in unison with a particular tuning fork. When a beaker containing water is brought under the block so that the block is completely dipped into the beaker, the string vibrates in its eleventh harmonic. Find the density of the material of the block.



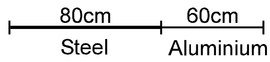
7. A wire of 9.8×10^{-3} kg mass per meter passes over a frictionless pulley fixed on the top of an inclined frictionless plane which makes an angle of 30° with the horizontal. Masses M_1 & M_2 are tied at the two ends of the wire. The mass M_1 rests on the plane and the mass M_2 hangs freely vertically downwards.

The whole system is in equilibrium. Now a transverse wave propagates along the wire with a velocity of 100 m/sec. Find the value of masses M_1 & M_2.

8. A uniform horizontal rod of length 40 cm and mass 1.2 kg is supported by two identical wires as shown in figure. Where should a mass of 4.8 kg be placed on the rod so that the same tuning fork may excite the wire on left into its fundamental vibrations and that on right into its first overtone? Take g = 10 m/s^2.



9. Figure shows an aluminium wire of length 60 cm joined to a steel wire of length 80 cm and stretched between two fixed supports. The tension produced in 40 N. The cross-sectional area of the steel wire is 1.0 mm^2 and that of the aluminium wire is 3.0 mm^2. What could be the minimum frequency of a tuning fork which can produce standing waves in the system with the joint as a node? The density of aluminium is 2.6 g/cm^3 and that of steel is 7.8 g/cm^3.



EXERCISE-5

1. A transverse wave is described by the equation $y = x_0 \cos 2\pi (vt - x/\lambda)$. The maximum particle velocity is two times the wave velocity provided λ = ... **[JEE - 96]**

2. A linearly polarised transverse wave is propagating in z-direction through a fixed point P in space. At time t_0, the x-component E_x and the y-component E_y of the displacement at P are 3 and 4 units respectively. At a later time t_1, if E_x at P is 2 units, the value of E_y will be **[REE - 96]**
(A) 5 units (B) 8/3 units (C) 3/8 units (D) 1/3 units

3. A travelling in a stretched string is described by the equation $y = A \sin (kx - \omega t)$. The maximum particle velocity is **[JEE - 97,1]**
(A) $A\omega$ (B) ω/k (C) $d\omega/dk$ (D) x/t

4. A place progressive wave of frequency 25 Hz, amplitude 2.5×10^{-5} m & initial phase zero propagates along the (–ve) x–direction with a velocity of 300 m/s. At any instant, the phase difference between the oscillations at two points 6 m apart along the line of propagation is __ & the corresponding amplitude difference is ____ m. **[JEE - 97, 2]**

5. The fundamental frequency of a sonometer wire increases by 6 Hz if its tension is increased by 44% keeping the length constant. Find the change in the fundamental frequency of the sonometer when the length of the wire is increased by 20% keeping the original tension in the wire. **[JEE - 97, 5]**

6. The equation of transverse wave in a vibrating string is $y = 0.021 \sin (x + 30t)$, where the distances are in meter and time is in second. If the linear density of the string is 1.3×10^{-4} kg/m, then the tension in the string in newton will be **[JEE - 97]**
(A) 10 (B) 0.5 (C) 1 (D) 0.117

7. Out of the following three wave forms ; **[REE - 97,1]**
(a) $2A \cos kx \sin \omega t$
(b) $2A \cos (\Delta\omega/2)\, t \cos (\omega t - kx)$ &
(c) $2A \cos (\phi/2) \sin (\omega t - kx + \theta)$ _____ represent the phenomenon of stationary wave.

8. When the tension in a stretched string is quadrupled, the velocity of the transverse wave is
(A) greater than twice the original velocity (B) twice the original velocity
(C) less than twice the original velocity (D) not changed **[REE - 97]**

9. The (x , y) co–ordinates of the corners of a square plate are (0, 0) (L, 0) (L, L) & (0, L). The edges of the plate are clamped & transverse standing waves are set up in it. If u (x, y) denotes the displacement of the plate at the point (x, y) at some instant of time, the possible expression(s) for u is/are :
(a = positive constant) **[JEE - 98,2]**

(A) $a \cos\left(\frac{\pi x}{2L}\right) \cos\left(\frac{\pi y}{2L}\right)$ (B) $a \sin\left(\frac{\pi x}{L}\right) \sin\left(\frac{\pi y}{L}\right)$

(C) $a \sin\left(\frac{\pi x}{L}\right) \sin\left(\frac{2\pi y}{L}\right)$ (D) $a \cos\left(\frac{2\pi x}{L}\right) \sin\left(\frac{\pi y}{L}\right)$

10. A string of length 0.4 m & mass 10^{-2} kg is tightly clamped at its ends. The tension in the string is 1.6N. Identical wave pulses are produced at one end at equal intervals of time, Δt. The minimum value of Δt which allows constructive interference between successive pulses is **[JEE - 98, 2]**
(A) 0.05 s (B) 0.10 s (C) 0.20 s (D) 0.40 s

11. A transverse sinusoidal wave of amplitude a, wavelength λ & frequency f is travelling on a stretched string. The maximum speed of any point on the string is v/10, where v is speed of propagation of the wave. If $a = 10^{-3}$ m & $v = 10\ ms^{-1}$, then λ & f are given by **[JEE - 98]**

(A) $\lambda = 2\pi \times 10^{-2}$ m (B) $\lambda = 10^{-2}$ m (C) $f = \dfrac{10^3}{2\pi}$ Hz (D) $f = 10^4$ Hz

12. The fundamental frequency of a sonometer wire increases by 6 Hz if its tension is increased by 44% keeping the length constant. Find the change in the frequency of the sonometer wire when the length of the wire is increased by 20% keeping the original tension in the wire. **[REE - 98]**

13. A cork floats on the water surface. A wave given by

$$y = 0.1 \sin 2\pi (0.1x - 2t)$$

passes over the water surface. Due to passage of the wave, the cork moves up and down. The maximum velocity of the cork, in ms^{-1}, is **[REE - 98]**

(A) 0.1 (B) 0.1π (C) $0.4\ \pi$ (D) π

14. A wave given by $\xi = 10 \sin [\,80\pi t - 4\pi x]$ propagates in a wire of length 1m fixed at both ends. If another wave is superimposed on this wave to produce a stationary wave then **[REE - 98]**

(A) the superimposed wave is $\xi = 10 \sin [80\pi t + 4\pi x]$
(B) the amplitude of the stationary wave is 0.5 m.
(C) the wave length of the stationary wave is 20.
(D) the number of total nodes produced in the wire are 3.

15. Which of the following parameters are required to specify completely a monochromatic plane wave in vacuum? **[REE - 98]**

(A) Amplitude (B) Frequency (C) Initial phase (D) state of polarization

16. In hydrogen spectrum the wavelength of H_α line is 656 nm, whereas in the spectrum of a distant galaxy, H_α line wavelength is 706 nm. Estimated speed of the galaxy with respect to earth is, **[JEE - 99, 2]**

(A) 2×10^8 m/s (B) 2×10^7 m/s (C) 2×10^6 m/s (D) 2×10^5 m/s

17. As a wave propagates :

(A) the wave intensity remains constant for a plane wave
(B) the wave intensity decreases as the inverse of the distance from the source for a spherical wave
(C) the wave intensity decreases as the inverse square of the distance from the source for a spherical wave
(D) total intensity of the spherical wave over the spherical surface centered at the source remains constant at all times . **[JEE - 99, 3]**

18*. $y(x, t) = 0.8/[(4x + 5t)^2 + 5]$ represents a moving pulse, where x & y are in meter and t in second. Then: **[JEE - 99, 3]**

(A) pulse is moving in +x direction (B) in 2s it will travel a distance of 2.5 m
(C) its maximum displacement is 0.16 m (D) it is a symmetric pulse .

19. In a wave motion $y = a \sin (kx - \omega t)$, y can represent : **[JEE - 99, 3]**

(A) electric field (B) magnetic field (C) displacement (D) pressure

20. Standing waves can be produced : **[JEE - 99, 3]**

(A) on a string clamped at both the ends
(B) on a string clamped at one end and free at the other
(C) when incident wave gets reflected from a wall
(D) when two identical waves with a phase difference of p αre moving in same direction

21. A long wire PQR is made by joining two wires PQ and QR of equal radii. PQ has length 4.8 m and mass 0.06 kg. QR has length 2.56 m and mass 0.2kg. The wire PQR is under a tension of 80N. A sinusoidal wave–pulse of amplitude 3.5cm is sent along the wire PQ from the end P. No power is dissipated during the propagation of the wave–pulse. Calculate **[JEE - 99, 4 + 6]**

(a) the time taken by the wave–pulse to reach the other end R of the wire, and
(b) the amplitude of the reflected and transmitted wave–pulses after the incident wave–pulse crosses the joint Q.

22. Two metallic strings A and B of different materials are connected in series forming a joint. The strings have similar cross–sectional area. The length of A is $l_A = 0.3$m and that B is $l_B = 0.75$m. One end of the combined string is tied with a support rigidly and the other end is loaded with a block of mass *m* passing over a frictionless pulley. Transverse waves are set up in the combined string using an external source of variable frequency. Calculate **[REE - 99]**

(i) the lowest frequency for which standing waves are observed such that the joint is a node and

(ii) the total number of anti–nodes at this frequency. The densities of A & B are $6.3 \times 10^3 kg\ m^{-3}$ and $2.8 \times 10^3 kg\ m^{-3}$ respectively.

23. Two vibrating strings of the same material but lengths L & 2 L have radii 2 r and r respectively. They are stretched under the same tension. Both the strings vibrate in their fundamental modes, the one of length L with frequency f_1 and the other with frequency f_2. The ratio f_1/f_2 is given by : **[JEE - 2000 Screening,1]**

(A) 2 (B) 4 (C) 8 (D) 1

24. A wave pulse starts propagating in the + x direction along a non–uniform wire of length 10 m with mass per unit length given by $m = m_0 + \alpha x$ and under a tension of 100 N . Find the time taken by the pulse to travel from the lighter end (x = 0) to the heavier end .

($m_0 = 10^{-2}$ kg/m and $\alpha = 9 \times 10^{-3}$ kg/m²) **[REE - 2000 Mains, 6]**

25. Two sinusoidal waves with same wavelengths and amplitude travel in opposite directions along a string with a speed 10 ms⁻¹. If the minimum time interval between instants when the string is flat is 0.5s, the wavelength of the waves is : **[REE - 2000]**

(A) 25 m (B) 20 m (C) 15 m (D) 10 m

26. A longitudinal travelling wave transports **[REE - 2000]**

(A) energy and linear momentum (B) energy and angular momentum

(C) energy and torque (D) angular momentum and torque

27. The intensity of a progressing plane wave in loss-free medium is

(A) directly proportional to the square of amplitude of the wave

(B) directly proportional to the velocity of the wave

(C) directly proportional to the square of frequency of the wave

(D) inversely proportional to the density of the medium. **[REE - 2000]**

28. The ends of a stretched wire of length L are fixed at x = 0 & x = L . In one experiment the displacement of the wire is $y_1 = A \sin(\pi x/L) \sin \omega t$ & energy is E_1 and in other experiment its displacement is $y_2 = A \sin(2\pi x/L) \sin 2\omega t$ and energy is E_2 . Then : **[JEE - 2001 Screening, 2]**

(A) $E_2 = E_1$ (B) $E_2 = 2E_1$ (C) $E_2 = 4E_1$ (D) $E_2 = 16E_1$

29. Two pulses in a stretched string, whose centres are initially 8 cm apart, are moving towards each other as shown in the figure. The speed of each pulse is 2 cm/s. After 2 seconds, the total energy of the pulses will be : **[JEE - 2001 Screening, 2]**

(A) zero (B) purely kinetic

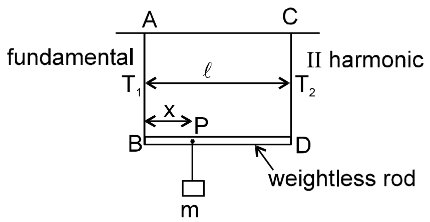
(C) purely potential (D) partly kinetic and partly potential

30. A sonometer wire resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire . When this mass is replaced by a mass M, the wire resonates with the same tuning fork forming three antinodes for the same positions of the bridges. The value of M is **[JEE - 2002 Screening, 3]**

(A) 25 kg (B) 5 kg (C) 12.5 kg (D) 1/25 kg

31. A string of mass ' m ' and length ℓ, fixed at both ends is vibrating in its fundamental mode.The maximum amplitude is ' a ' and the tension in the string is ' T '. Find the energy of vibrations of the string. **[JEE - 2003 ,4]**

32. A massless rod BD is suspended by two identical massless strings AB and CD of equal lengths. A block of mass 'm' is suspended point P such that BP is equal to 'x', if the .fundamental frequency of the left wire is twice the fundamental frequency of right wire, then the value of x is : **[JEE - 2006 ,3/184]**



(A) $\ell/5$ (B) $\ell/4$ (C) $4\ell/5$ (D) $3\ell/4$

ANSWER

EXERCISE - 1

SECTION (A) :

A 1. (a) 3.33i m/s (b) –5.48 cm
(c) 0.667 m, 5.00 Hz (d) 11.0 m/s

A 2. (a) 31.4 rad/s (b) 1.57 rad/m
(c) y = (0.120 m) sin (1.57 x – 31.4t)
(d) 3.77 m/s (e) 118 m/s^2

A 3. (a) y = (8.00 cm) sin (7.85x + 6πt)
(b) y = (8.00 cm) sin (7.85x + 6πt – 0.785)

A 4. $0.5 \sin\left(\frac{\pi}{3}t - \frac{\pi}{18}x + \frac{7\pi}{9}\right)$

A 5. (a) D, E, F (b) A, B, H (c) C, G (d) A, E

SECTION (B) :

B 1. 520 m/s **B 2.** 0.02 s

SECTION (C) :

C 1. (a) y = (7.50 cm) sin (4.19x – 314t) (b) 625 W
C 2. (a) 0.47 W (b) 9.4 mJ

SECTION (D) :

D 1. (a) Zero (b) 0.300 m.
D 2. 1.41 y_m
D 3. (a) 0.52 m; (b) 40 m/s ; (c) 0.40 m

SECTION (E) :

E 1. (a) 7.91 Hz; (b) 15.8 Hz; (c) 23.7 Hz.
E 2. (a) 144 m/s; (b) 60.0 cm; (c) 241 Hz
E 3. (a) 105 Hz; (b) 158 m/s
E 4. (a) 0.25 cm (b) 1.2 × 10^2 cm/s; (c) 3.0 cm; (d) 0
E 5. 36 N
E 6. (a) 100 Hz (b) 700 Hz
E 7. (i) y_1 = 1.5 cos {(π/20)x – 72πt},
y_2 =1.5 cos {(π/20)x + 72 πt}
(ii) 10 , 30 , 50 cm and 0 , 20 , 40 , 60 cm
(iii) 0
E 8. 300 Hz

EXERCISE - 2

SECTION (A) :

A 1. All **A 2.** B **A 3.** A **A 4.** A
A 5. B **A 6.** A **A 7.** C. **A 8.** C
A 9. B

SECTION (B) :

B 1. D **B 2.** D **B 3.** C **B 4.** A

SECTION (C) :

C 1. C **C 2.** A **C 3.** A **C 4.** D

SECTION (D) :

D 1. B **D 2.** B **D 3.** C **D 4.** D

EXERCISE - 3

1.	A	**2.**	AD	**4.**	B	**5.**	B
6.	C	**7.**	D	**8.**	D	**9.**	D
10.	C	**11.**	B	**12.**	A	**13.**	C
14.	B	**15.**	D	**16.**	B	**17.**	B
18.	C	**19.**	A	**20.**	C		

EXERCISE - 4

1. (a) 1.2 π m/s = 3.768 m/s (b) 10.8 π N
(c) 0 (d) 12.96 π^2
(e) 0 (f) 0 (g) 0.5 cm

2. (a) λ = 1 m , f = 400 Hz
(b) y = (3 mm) sin 2 πx cos 800 πt
(c) y_1 = (1.5 mm) sin (2 πx + 800 πt) ;
y_2 = (1.5 mm) sin (2 πx – 800 πt)
(d) KE_{max} = (1/4) mω^2 A^2 = 1.44 J

3. a_{max} = 5 mm; to the third overtone.

4. 1000/3Hz

5. (a) 30 Hz (b) 3rd, 5th and 7th
(c) 2nd, 4th, 6th (d) 48 m/sec.

6. 5.8 × 10^3 kg/m^3

7. m_2= 10 kg , m_1 = 20 kg

8. 5 cm from the left end **9.** 180 Hz

EXERCISE - 5

1. πx_0 **2.** B **3.** A **4.** π rad , 0 m

5. $\frac{6}{\sqrt{12}}$ = 5.48 Hz decrease

6. D **7.** (a) **8.** B **9.** BC

10. B **11.** AC **12.** 5 Hz. **13.** C

14. ABC **15.** ABC **16.** B **17.** ACD
18. BCD **19.** ABC **20.** ABC

21. (a) Time = 140 ms (b) $A_r = \frac{V_2 - V_1}{V_2 + V_1} A_i$ = 1.5 cm;

$A_t = \frac{2V_2}{V_1 + V_2} A_i$ = 2 cm

22. $\frac{5}{3}\sqrt{\frac{m}{70\,S}}$,
where S= area of cross section of wire, 8

23. D

24. $\frac{1}{15\,\alpha}\,[(m_0 + a\,l)^{3/2} - (m_0)^{3/2}] = \frac{10\sqrt{10} - 1}{105}$ s

25. D **26.** A **27.** ABC **28.** C

29. B **30.** A **31.** $\frac{\pi^2 a^2 T}{4L}$ **32.** A